EVALUATION OF SECTORAL FOREIGN TRADE ELASTICITIES OF AZERBAIJAN

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ABSTRACT

This study investigates the foreign trade elasticities of the oil, non-oil, and services sectors of the Azerbaijani economy, aiming to understand how consumers substitute between imported and domestic goods and how producers allocate output between domestic and foreign markets. CES-type Armington and Constant Elasticity of Transformation (CET) functions are estimated using the Marquardt nonlinear optimization method in Mathcad, providing a consistent framework for capturing substitution and transformation behaviors across sectors. The results reveal substantial intersectoral differences in both demand-side and supply-side elasticities, reflecting structural characteristics that shape the country's trade patterns. Although the estimated elasticities are derived under the assumption of stable behavioral relationships over time, the analysis offers valuable implications for policy assessment. In particular, the elasticities can be directly used in the calibration of general equilibrium and trade models designed to evaluate alternative policy scenarios. The study provides original sector-specific empirical estimates for Azerbaijan, contributing to a better understanding of foreign trade responses and supporting model-based policy design.

Keywords: Armington function, CET function, oil-rich economy, import, export.

JEL Classification: C5, F1

INTRODUCTION

Imports and exports are key indicators of an economy, and their volumes vary depending on prices and demand. The responsiveness of import and export volumes to relative price changes is measured by trade elasticity, which provides important insights into a country's foreign trade performance. Export elasticity describes how exporters respond to changes in various factors. When demand changes, the resulting dynamics in domestic and imported product volumes are captured by the price elasticity of imports (Imbs, J.; Mejean, I. (2017)). These elasticities are crucial for understanding the role of international prices in balancing trade, the optimal level of international portfolio diversification, the effects of regional trade agreements, and the welfare gains from expanding world trade (Feenstra, R.; Luck, P; Obstfeld, M; Russ, K. (2018)). General equilibrium models frequently employ Armington and Constant Elasticity of Transformation (CET) functions to represent international trade (Lofgren, H.; Cicowiez, M. (2018)).

The CET function models the producer's decision of whether to sell in the domestic or foreign market, while the Armington function captures consumers' choices between domestic and imported products. The Armington function is a CES-type function, named after Paul Armington, who introduced it for this purpose (Armington, P. (1969)). In a 1968 article, two Australian economists Powell, A. and Gruen, F. (1968) proposed the concept of constant elasticity of transformation.

Paul Armington claims that domestic and imported goods are not perfect substitutes. Consumers differentiate between them. According to Armington, total demand for good i is split between domestic and imported varieties. To estimate this split he employs a constant elasticity of substitution (CES) utility function depicted in Equation (1) (Annabi, N.; Cockburn, J.; Decaluwé, B. (2006); Armington, P. (1969)).

$$Q_i = \gamma_i(\delta m_i M_i^{\eta_i} + \delta d_i D_i^{\eta_i})^{\frac{1}{\eta_i}} \tag{1}$$

whereby γ_i is a parameter representing the effectiveness of substituting imported and domestic products for the i-th commodity or service group, δm_i and δd_i are CES distribution parameters, and η_i is used to calculate the elasticity of substitution between imported and domestic products. The elasticity of substitution between imported and domestic goods is then computed as $\sigma_i = \frac{1}{1+\eta_i}$.

According to profit-maximization behavior, for each group of goods and services, profit is maximized when the difference between total revenue and the combined cost of domestic and imported goods reaches its maximum (Hosoe, N.; Gasawa, K.; Hashimoto, H. (2021)).

$$\max \pi_i^q = P_i^q Q_i - \left[(1+\tau_i^m)P_i^m M_i + P_i^d D_i\right] \tag{2}$$

whereby, Q_i denotes the total demand in the country for the i-th group of goods and services, and P_i^q is the price per unit of products or services in that group. M_i – represents the quantity of imports for the i-th group, P_i^m is the import price, and τ_i^m is the tariff rate. D_i denotes the quantity of domestic production sold in the local market for the i-th group, and P_i^d is its price.

Thus, for various groups of goods and services in the economy, we consider the problem of maximizing the objective function (2) subject to constraint (1). This is a conditional optimization problem that can be solved using the method of Lagrange multipliers (Hosoe, N.; Gasawa, K.; Hashimoto, H. (2021)). Solving this problem allows us to estimate the portion of total demand for each type of goods and services that should be met by domestic production and the portion that should be fulfilled by imported products. From the first-order condition of this optimization problem, the ratio of imported to domestic products can be expressed as follows (Annabi, N.; Cockburn, J.; Decaluwé, B. (2006)):

$$\frac{M_i}{D_i} = \left(\frac{P_i^d}{P_i^m} \cdot \frac{\delta m_i}{\delta d_i}\right)^{\sigma_i} \tag{3}$$

Apparently, this ratio depends on the prices of imported and domestic products, distribution parameters, and the elasticity coefficient. The higher the elasticity, the more sensitive the import-to-domestic product ratio is to changes in the price ratio. In other words, even a small change in the price of imported or domestic products can lead to a substantial change in consumer demand. Conversely, when elasticity is low, even large changes in the price ratio result in only minor adjustments to the import-to-domestic product ratio.

Similarly, the producer can choose to sell products in the domestic market or export them. In doing so, the producer maximizes the following objective function to sell D_i of the total production Z_i in the domestic market and to export the volume E_i (Hosoe, N.; Gasawa, K.; Hashimoto, H. (2021)):

$$\max \pi_i = P_i^e E_i - \left[(1+\tau_i^z)P_i^z Z_i + P_i^d D_i\right] \tag{4}$$

In the i-th sector, the allocation of total production between exports and the domestic market is represented using a CES-type function known as the CET (Constant Elasticity of Transformation) function (Annabi, N.; Cockburn, J.; Decaluwé, B. (2006)).

$$Z_i = \theta_i(\xi e_i E_i^{\varphi_i} + \xi d_i D_i^{\varphi_i})^{\frac{1}{\varphi_i}} \tag{5}$$

whereby, θ_i is a parameter representing the efficiency of substituting quantities of products sold in the export and domestic markets for the i-th production sector, ξe_i and ξd_i are CET distribution parameters, and φ_i is used to calculate the elasticity of substitution between products sold in the export and domestic markets. The elasticity of substitution between exported and domestic products is then computed as $\sigma_i = \frac{1}{1+\varphi_i}$.

As shown, this is also a conditional optimization problem, and solving it using the method of Lagrange multipliers allows us to determine the portion of the i-th production sector's output that should be sold in the domestic market and the portion that should be exported. From this solution, the relative volume of exported to domestic products can be expressed as follows (Annabi et al., 2006):

$$\frac{D_i}{E_i} = \left(\frac{P_i^e}{P_i^d} \cdot \frac{\xi d_i}{\xi e_i}\right)^{\sigma_i} \tag{6}$$

Foreign trade elasticities provide crucial information about a country's economy and the behavior of consumers and producers, making their estimation an important input for building general equilibrium and various trade models. Many resource-rich countries are known to be dependent on resource prices (Shahbaz, M.; Destek, M; Okumus, I; Sinha, A. (2019); Guan, L.; Zhang, W.; Ahmad, F.; Naqvi, B. (2021)), including cases where high import volumes result from the reduced competitiveness of other sectors—a manifestation of the resource curse (Mikesell, R. (1997); Auty, R. (2014)). Oil- and gas-rich Azerbaijan also faces challenges due to the large share of the oil sector in its exports, which makes the economy dependent on oil (Czech, K. (2018); Sadik-Zada, E. (2019); Sadik-Zada, E.; Gatto, A. (2021); Sadik-Zada, E.; Loewenstein, W.; Hasanli, Y. (2021)), as well as the substantial volume of imports in the non-oil sector (Seyfullayev, I. (2023)). In recent years, Azerbaijani government has pursued policies aimed at reducing the economy's dependence on commodity revenues and promoting diversification of exports (Aliyev, U.; Guliyeva, G. (2025); Dadashov, O., (2023)). In this context, a strategic roadmap has been developed, including a development strategy for 2016–2020, a long-term vision for 2025, and a target vision for the period beyond 2025.

The main objectives of this roadmap are to diversify Azerbaijan's economy, strengthen competitiveness, reduce dependence on oil, increase the size and quality of non-oil sector exports, and further improve employment levels and the welfare of the population. During the decision-making process, evaluating elasticity parameters that reflect the current state of the economy can help in developing different trade policy scenarios. From this perspective, assessing foreign trade elasticities by dividing the economy into oil, non-oil, and service sectors is particularly important. The findings could have relevance for applied economic policies of Azerbaijan. Moreover, econometric results could contribute to the completion of the mosaic of a assessment of the policy options for Azerbaijan through their integration with the holistic perspectives of the general equilibrium and trade models of Azerbaijan that are widely employed by researchers and government bodies, such as the Central Bank of Azerbaijan.

LITERATURE REVIEW

A large number of studies in the literature focus on the estimation of Armington and CET functions, employing various methods to estimate these elasticities. Examining the results, it is evident that these studies have been conducted for different countries and at various sectoral levels. For example, Saikkonen, L. (2015) estimates Armington elasticities for multiple sectors of the South African economy using both linear and non-linear least squares methods, with elasticity values ranging from 0.386 to 1.379. Ntombela, S., Kalaba, M. and Bohlmann, H. (2018) estimated elasticities using linear least squares (LS) for South African agricultural products and concluded that, while agriculture in aggregate is inelastic, individual products show responsiveness to price changes. For most products, Armington estimates were close to unity, indicating that agricultural imports are not perfect substitutes for domestic goods. Moreover, export supply elasticities for grains were higher than those for fruit and meat, suggesting that domestic grain production is more sensitive to changes in export market prices. Delahaye, E. and Milot, C. (2020) criticize the practice of using identical elasticities for all countries in AGE models of international trade, such as the Global Trade Analysis Project (GTAP). They estimate Armington elasticities for the UK across various goods and services in agriculture, manufacturing, and services, finding values between 0.01 and 2.69. These results differ from the parameter values typically used in the model, highlighting the importance of estimating elasticities for each country individually.

Bajzik, J.; Havranek, K., Irsova, Z. and Schwarz, J. (2019) analyze 3,524 estimates of Armington elasticities and investigate the sources of variation among them. The study finds that data frequency is a major factor contributing to these differences, with estimates derived from less frequent data tending to be smaller. The authors also note that estimates based on cross-sectional data are generally larger than those based on time-series data. Blonigen, B. and Wilson, W. (1999) also estimate Armington elasticities for various sectors of the U.S. economy and investigate the reasons for differences in elasticity values across sectors. The study finds that the presence of foreign-owned affiliates significantly affects flexibility, as multinational companies in a sector blur the distinction between domestic and imported products. Another factor influencing the elasticity of substitution is the presence of entry barriers in the sector, which reduces the substitutability between domestic and imported products. Olekseyuk, Z. and Schürenberg-Frosch, H. (2016) emphasize the importance of Armington elasticities in general equilibrium models and the sensitivity of model results to the choice of elasticity. They note that using elasticities from other countries when constructing general equilibrium models may lead to inaccurate results. Accordingly, it is recommended to estimate these elasticities for each country and sector whenever possible; if this is not feasible, multiple assessments using different elasticity values should be conducted. Furthermore, by employing cointegration and panel fixed-effects analyses, the study evaluates the first-order condition for various European countries and demonstrates that the results differ across countries. Although most evaluations in the literature employ econometric methods, such approaches require a sufficient number of observations, which may not be feasible in developing countries to obtain statistically significant results. In this context, Arndt, C.; Robinson, S. and Tarp, F. (2002) developed the maximum entropy method and estimated Armington and CET elasticities for the Mozambican economy using this approach. Armington elasticities ranged from 0.57 to 5.54 across different sectors, while CET elasticities ranged from 0.33 to 2.84. Ahmad, S.; Montgomery, C. and Schreiber, S. (2021) highlight sectoral differences as a source of variation in existing studies, noting that different levels of aggregation yield different results. Consequently, estimates for more disaggregated sectors were higher than those for aggregated sectors in most studies.

A recent World Bank study (Devarajan, S.; Go, D.; Robinson, S. (2023)) highlights the scarcity of elasticity estimates in the literature, particularly for developing countries. Using a vector error correction model, the study estimates Armington and CET elasticities for 191 countries. On average, both Armington and CET elasticities are 1.4 for developed countries, while Armington elasticities are 0.7 and CET elasticities are 0.6 for developing countries. The study notes that, generally, the lower elasticities in developing countries reflect their limited ability to respond adequately to various price changes. For Azerbaijan, the estimated aggregate elasticities are 0.503 for imports and 0.362 for exports. This represents the only evaluation of the Azerbaijani economy identified in the literature review, and sectoral-level elasticities have not yet been estimated for the country.

Ahmad, S.; Montgomery, C.; Schreiber, S. (2021) review existing studies, summarize the methods used to estimate Armington elasticities, and compare the results obtained. The study considers the mathematical and methodological foundations of the import price method, system of equations method, trade cost method, and markup method for assessing Armington elasticities. Annabi, N.; Cockburn, J.; Decaluwé, B. (2006) focus on the estimation of functional forms and their parameters used in general equilibrium models, the mathematical and methodological foundations of CES-type function parameter estimation — including Armington and CET functions—and information on the methods employed in existing research. In practice, when constructing general equilibrium models, trade elasticities are estimated using econometric or entropy methods. In some cases, researchers rely on elasticity values from other countries available in literature, or occasionally on their own judgment. Although econometric methods are the most widely used, they require the availability of relevant indicators for a given country and their necessary dynamics. The econometric approach involves evaluating the first-order condition of the optimization problem using the linear least squares method or estimating the parameters of the CES function using the nonlinear least squares method.

In our study, Armington and CET elasticities are estimated for the oil, non-oil, and service sectors. As observed in the literature review, when data are available, econometric methods are the most commonly used approach for estimating elasticities. The first-order condition obtained from solving the optimization problem is convenient for econometric evaluation using linear least squares (LS). However, due to difficulties in obtaining the necessary sectoral-level price data, we will estimate CES-type Armington and CET functions using the nonlinear least squares method.

METHODOLOGY

Although the CES function has a more general structure and allows for the consideration of various aspects of economic agents' behavior, its nonlinearity - even after logarithmization - prevents evaluation using the linear least squares method. Therefore, the nonlinear least squares method is employed to estimate this function (Kubaniva, M.; Tabata, M.; Hasebe, Y. (1991)).

Assume that the theoretical form of a nonlinear function F, which characterizes the dependence of the dependent variable Y on the explanatory variables $X_1, X_2, \ldots, X_n$, is known:

$$Y = F(X_1, X_2, \ldots, X_n)$$

However, the values of the parameters $a_1, a_2, \ldots, a_n$ associated with the explanatory variables $X_1, X_2, \ldots, X_n$ are unknown. Each parameter a_i reflects the effect of the explanatory variable X_i on the dependent variable Y. These parameters must therefore be estimated. For this purpose, m observations are collected. For each observed value

Y_i, the corresponding values of the explanatory variables $(X_{i1}, X_{i2}, \dots, X_{in})$ for $i = 1,2,\dots,m$ are obtained. Thus,

$$Y_i = F_i\,(a_1, a_2, \dots, a_n; X_{i1}, X_{i2}, \dots, X_{in}) + U_i, \quad i = \overline{1,m}, \tag{7}$$

where U_i denotes the disturbance term. The objective in (7) is to identify values of the parameters $a_1, a_2, \dots, a_n$ such that the theoretical values of the dependent variable are as close as possible to the observed values. In other words, the deviations U_i must be minimized. The parameters satisfying this condition are typically estimated using the method of least squares.

The Armington and CET functions employed in our study are nonlinear with respect to their parameters, similar to the CES production function. It should be noted that if a function is nonlinear in variables (but linear in parameters), linearization is straightforward. Because statistical values of the variables are drawn from observations, the function can be linearized regardless of the specific type of nonlinearity. For example, consider the Cobb–Douglas production function:

$$Y = AK^aL^b \ ,$$

where Y denotes GDP, K capital, and L labor, while A, a, and b are the parameters. Taking logarithms of both sides yields:

$$\log(Y) = \log(A) + a\log(K) + b\log(L)$$

By defining $\log(Y) = Y^*$, $\log(A) = A^*$, $\log(K) = K^*$, and $\log(L) = L^*$, the model becomes a linear specification:

$$Y^* = A^* + aK^* + bL^*.$$

In any applied econometric software package (such as EViews or SPSS), the parameters of such linear regression models can be estimated using various methods, including ordinary least squares (OLS). The Gauss–Markov assumptions and the Gauss–Markov theorem apply to regression models that are linear—in other words, linear in parameters. Although certain extensions of the Gauss–Markov framework exist, there is no general theorem that guarantees the Gauss–Markov conditions or BLUE-type optimality for regression models that are nonlinear in parameters (Verbeek, M., (2017). Consequently, for nonlinear specifications such as the CES function, numerical estimation methods remain the standard and accepted approach in applied research.

Therefore, for nonlinear-in-parameter models, the minimization of the objective function

$$S(a_1, a_2, \dots, a_n) = U_1^2 + U_2^2 + \cdots + U_n^2 = \sum_{i=1}^{n} U_i^2 \rightarrow min$$

must be performed using alternative techniques. Since the objective function S is nonlinear in parameters, applying Fermat's theorem becomes impractical. Taking partial derivatives with respect to the parameters, setting them equal to zero, and solving the resulting system of equations is often highly complex or even infeasible. Thus, the minimization problem is typically addressed using approximate and numerical procedures.

Among numerical algorithms for minimizing S are the Marquardt method (a modification of the Newton–Gauss algorithm), Powell's version of the least squares method, the hybrid method, and others. In this study, we employ the Marquardt algorithm (Marquardt, 1963), which is recognized as a modification of the Newton–Gauss method (Björck, 1996), Powell's least squares approach, the hybrid method, and the method introduced by Levenberg (Levenberg, 1944).

In our study, Armington and CET functions were evaluated using the Marquardt method based on a program specifically developed for this study in Mathcad (Hasanli, Y.; Sadik-Zada, E.; Ismayilova, S.; Rahimli, G.; Ismayilova, F. (2023)).

DATA COLLECTİON AND PROCESSİNG

As mentioned, the Armington function enables the estimation of how imported and domestically produced products substitute for one another. We have estimated the Armington function for the Azerbaijan economy across the oil, non-oil and service sectors. For this purpose, the dependence of total demand for the products of each commodity or service group on the quantities of domestic and imported products in that group was evaluated econometrically.

The statistical data used for the econometric evaluation were collected from the Balance of Payments and the System of National Accounts (SNA) for 2009–2021. Accordingly, from the Balance of Payments tables, the imports of the oil and gas sector were considered as the imports of the oil sector, while imports of other sectors were considered as the imports of the non-oil sector. For the imports of the service sector, the receipts from the balance of services in the Balance of Payments were used. To determine the volume of domestic production sold on the local market, the value of exports was subtracted from the total output of each sector. Export data were taken from the Balance of Payments, while total output was obtained from the total output tables for types of economic activities and aggregated across the three sectors considered in the study. Additionally, the indicators obtained from the Balance of Payments were converted into manat using the exchange rate series for the respective years. The total demand for each group of goods and services across the country is defined as the sum of the value of domestically produced products sold on the local market and the value of imported products. Imports by sector are denoted by M, the value of domestic products sold on the local market by D, and the total demand for goods and services in each sector by Q. Statistical data used to estimate the parameters of the Armington functions for the aforementioned sectors are presented in Table 1. It should be noted that all indicators expressed as percentages were entered into the program developed in Mathcad. Based on this program, the parameters of the Armington function were estimated, and possible adequacy tests were conducted.

Table 1. Database for estimating the Armington functions

	Oil sector			**Non-oil sector**			**Services setor**		
	Q (mln manat)	M (mln manat)	D (mln manat)	Q (mln manat)	M (mln manat)	D (mln manat)	Q (mln manat)	M (mln manat)	D (mln manat)
2009	1021.8	560.1	461.7	19892.1	4651.0	15241.1	17776.5	2711.1	15065.4
2010	1449.0	670.1	778.9	22358.0	4726.4	17631.6	19592.5	3038.3	16554.3
2011	1806.4	896.4	909.9	28156.8	7135.1	21021.7	23545.4	4515.1	19030.3
2012	2472.8	813.4	1659.3	31940.1	7312.2	24627.9	26295.4	5619.6	20675.8
2013	2182.1	910.0	1272.1	35953.1	7791.4	28161.7	30214.8	6489.5	23725.3
2014	2346.6	1121.6	1224.9	36625.7	6157.3	30468.4	34325.4	8101.6	26223.8
2015	4652.5	3748.7	903.8	39256.5	11498.2	27758.3	39062.4	13529.6	25532.7
2016	5324.8	4121.8	1203.0	40717.3	11815.6	28901.7	41098.5	13315.4	27783.2
2017	7067.8	2282.3	4785.5	44224.9	13081.2	31143.7	46020.3	13715.2	32305.1
2018	5295.1	2967.1	2328.0	47269.3	15652.0	31617.3	46157.0	11479.7	34677.3
2019	5394.6	3198.6	2196.1	49910.9	16071.5	33839.4	50217.1	10841.1	39376.0
2020	7590.5	3189.3	4401.2	47607.2	13940.8	33666.3	51364.2	9284.1	42080.1
2021	6443.2	2762.0	3681.2	52643.2	14949.7	37693.5	57094.0	10061.1	47032.9

Source: Data from the State Statistical Committee of Azerbaijan and the authors' calculations

As mentioned, the CET function reflects how total output is allocated between the domestic and foreign markets. In this study, the CET function was estimated for the oil, non-oil, and service sectors. For this purpose, total output in these sectors was treated as the dependent variable, while the volumes of output sold on the domestic market and exported were treated as the independent variables. Total output by type of economic activity was aggregated across the three sectors to serve as the total output indicator. Export data were obtained from the Balance of Payments and converted into manat using the exchange rate. To determine the volume of domestic production sold on the local market, exports from each sector were subtracted from total output. Consequently, the research database was formed as shown in Table 2.

Total output is denoted by Y, exports by E, and the volume of domestic production sold on the local market by D.

Table 2. Database for estimating the CET functions

	Oil sector			Non-oil sector			Services		
	Y (mln manat)	E (mln manat)	D (mln manat)	Y (mln manat)	E (mln manat)	D (mln manat)	Y (mln manat)	E (mln manat)	D (mln manat)
2009	16437.3	15975.6	461.7	16142.9	901.8	15241.1	16486.4	1421.0	15065.4
2010	20864.9	20086.0	778.9	18726.4	1094.8	17631.6	18206.2	1651.9	16554.3
2011	26878.0	25968.1	909.9	22304.5	1282.9	21021.7	21178.8	2148.5	19030.3
2012	25605.7	23946.4	1659.3	26136.0	1508.2	24627.9	24014.8	3339.1	20675.8
2013	24674.7	23402.6	1272.1	29544.7	1383.0	28161.7	26947.4	3222.1	23725.3
2014	21994.2	20769.3	1224.9	31741.6	1273.2	30468.4	29575.5	3351.8	26223.8
2015	16430.1	15526.4	903.8	30095.6	2337.3	27758.3	32465.3	6932.6	25532.7
2016	21221.3	20018.3	1203.0	30955.1	2053.4	28901.7	35515.1	7731.9	27783.2
2017	28039.9	23254.3	4785.5	33647.9	2504.2	31143.7	40275.5	7970.3	32305.1
2018	34927.2	32599.2	2328.0	34367.5	2750.2	31617.3	42651.5	7974.2	34677.3
2019	32825.3	30629.2	2196.1	36986.2	3146.8	33839.4	45770.5	6394.5	39376.0
2020	22790.5	18389.3	4401.2	36676.9	3010.5	33666.3	46535.1	4455.0	42080.1
2021	36227.6	32546.4	3681.2	42024.0	4330.5	37693.5	53485.1	6452.2	47032.9

Source: Data from the State Statistical Committee of Azerbaijan and the authors' calculations

ESTIMATION RESULTS

1.1. Estimation of the Armington function parameters

The estimation results of the Armington function for the oil sector are as follows:

$$Q_j = 1.06 \cdot (0.58 \cdot M_j^{-0.00007} + 0.42 \cdot D_j^{-0.00007})^{-15243}$$

R^2=0.98, DW=1.72

The coefficient of determination of the model is 0.98, indicating that 98% of the variation in total demand is explained by changes in the volume of domestically produced goods sold in the local market and imported products. The Durbin–Watson statistic of 1.72 suggests the absence of first-order autocorrelation.

Figure 1 presents the actual and fitted values of the demand for goods and services in the oil sector, along with the dynamics of the residuals between them.

Based on the model results, the elasticity of substitution between imported and domestically produced products can be calculated as follows:

$$\sigma_{oil} = \frac{1}{1+0.00007} = 0.99$$

As can be seen, the elasticity of substitution between domestic and imported products in the oil sector is very close to unity. This indicates that locally produced goods in this sector can effectively substitute imported goods. According to formula (3), the elasticity of mutual substitution between imported and domestic products in the oil sector is close to one, meaning that the ratio of the volume of imported goods sold in the country to the volume of domestic goods remains approximately constant under any price change. However, there is a slight tendency for the volume of domestic products to exceed that of imported products. Based on calculation using expression (3), a 1% increase in the import price results in a reduction of the ratio of imported to domestic products by approximately 0.98%.

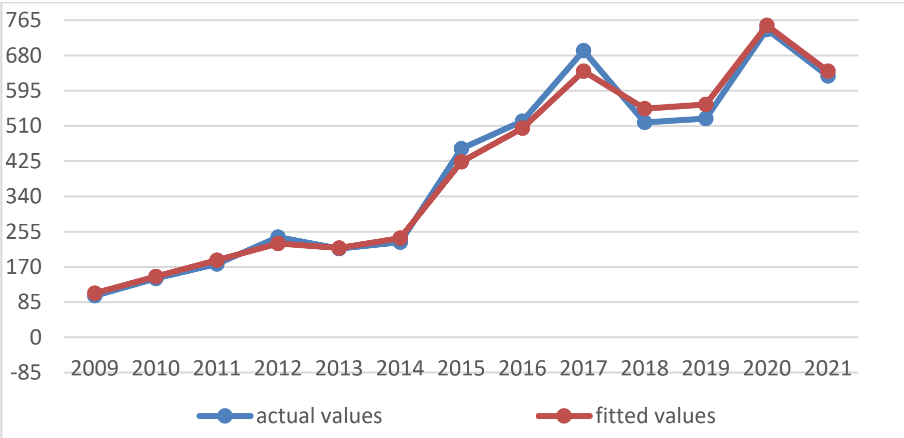

Figure 1. Actual and fitted values of the demand for goods and services in the oil sector

The estimation results of the Armington function for the non-oil sector are presented as follows:

$$Q_j = 1.002 \cdot (0.24 \cdot M_j^{0.7} + 0.76 \cdot D_j^{0.7})^{1.4}$$

R2=0.99, DW=1.88

The coefficient of determination indicates that 99% of the variation in demand for the non-oil sector is explained by changes in the volume of imports and domestically produced goods. The Durbin–Watson statistic, being close to 2, also suggests the absence of first-order autocorrelation in the residuals. Figure 2 illustrates the actual and fitted values of total demand for the non-oil sector, along with the dynamics of the residuals.

Based on the model results, the elasticity of substitution between imported and domestic products in the non-oil sector can be calculated as follows:

$$\sigma_{non-oil} = \frac{1}{1-0.7} = 3.3$$

As can be seen, the elasticity of substitution in the non-oil sector exceeds unity. According to the first-order condition, a 1% increase in the price of imported products results in an approximate **3.2% decrease** in the ratio of imported to domestic product volumes. This indicates that locally produced goods in the non-oil sector can substitute for imported goods. In other words, the potential output of the non-oil sector in the country is higher than its current production.

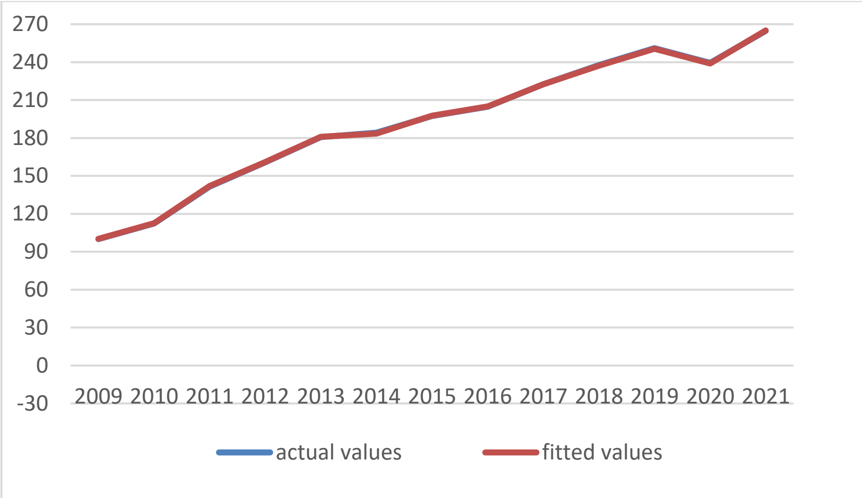

Figure 2. Actual and fitted values of the demand for goods and services in the non-oil sector

The estimation of the Armington function, which captures the dependence of total demand for services on the volume of imported and domestically produced products in the service sector, yielded the following results:

$$Y_j = 0.98 \cdot (0.25 \cdot M_j^{-0.00003} + 0.75 \cdot D_j^{-0.00003})^{-40563}$$

R2=0.99, DW=1.28

The high value of the coefficient of determination indicates that 99% of the variation in total demand for the service sector is explained by changes in the volume of imported and domestically produced products.

Figure 3 presents the actual and fitted values of total demand in the service sector, along with the dynamics of the residuals.

Based on the model results, the elasticity of substitution between domestic and imported products in the service sector was calculated as follows:

$$\sigma_{ser} = \frac{1}{1+0.00003} = 0.99$$

This elasticity of substitution also indicates that the elasticity between domestic and imported products in the service sector is close to unity. In other words, a 1% change in relative prices leads to an approximately proportional change in the ratio of imported to domestic products.

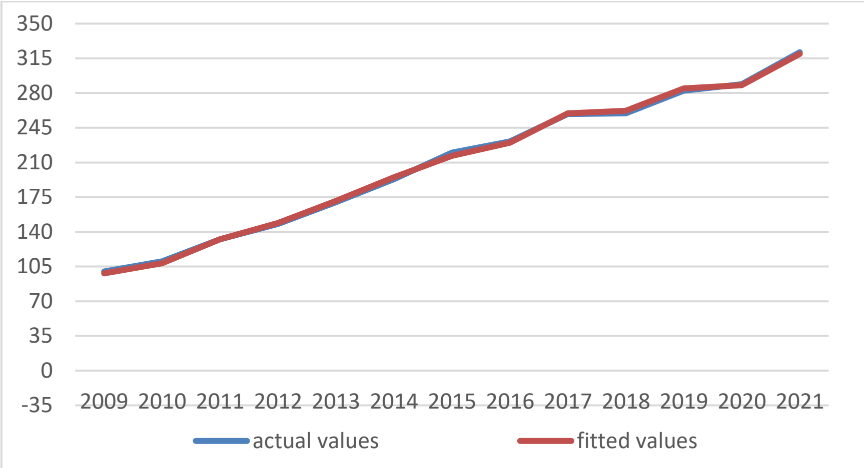

Figure 3. Actual and fitted values of total demand in the service sector

Estimation of the parameters of the CET function

The estimation of the CET function parameters for the oil sector yielded the following results:

$$Y_j = 0.97 \cdot (0.92 \cdot E_j^{0.14} + 0.08 \cdot D_j^{0.14})^{6.9}$$

R^2=0.99, DW=2.07

Figure 4 presents the actual and fitted values of total output in the oil sector, along with the dynamics of the residuals.

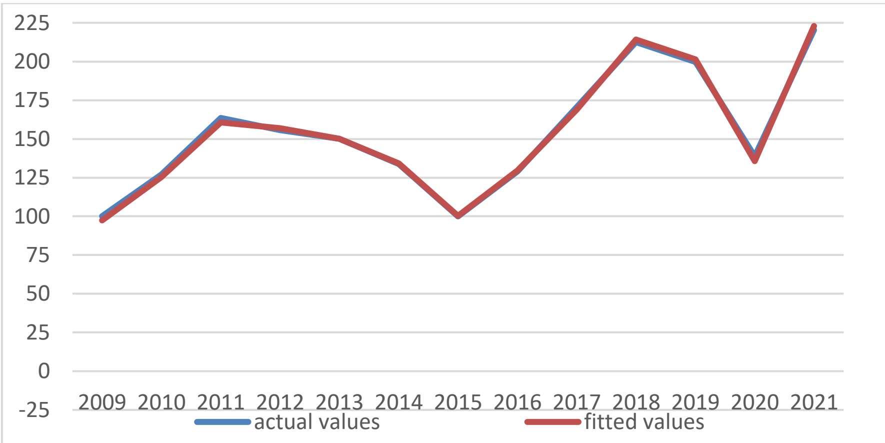

Figure 4. Actual and fitted values of total output in the oil sector

Based on the model results, the elasticity of substitution between products sold in the domestic and foreign markets in the oil sector was calculated as follows:

$$\sigma_{oil} = \frac{1}{1-0.14} = 1.17$$

As can be seen, the elasticity between domestically produced oil products sold on the local market and exported abroad is slightly greater than unity. This indicates that there is potential to increase exports of oil products currently sold domestically. According to formula (6), the sale of domestically produced oil products in foreign markets tends to increase relative to sales in the domestic market. More precisely, based on the first-order condition, a 1% increase in the foreign market price of goods and services in this sector raises the ratio of domestic-to-export sales of domestically produced goods and services in this sector by 1.17%.

The results of the assessment of the dependence of total output in the non-oil sector on the volumes sold in foreign and domestic markets are as follows:

$$Y_j = 1.001 \cdot (0.06 \cdot E_j^{0.4} + 0.94 \cdot D_j^{0.4})^{2.5}$$

R2=0.99, DW=1.41

Figure 5 presents the actual and fitted values of total output in the non-oil sector, along with the dynamics of the residuals.

Based on the model results, the elasticity of substitution between domestic and foreign markets for goods and services produced in the non-oil sector was calculated as follows:

$$\sigma_{non-oil} = \frac{1}{1-0.4} = 1.67$$

This result indicates that a 1% increase in the price of non-oil sector products in the foreign market leads to a 1.68% increase in the ratio of domestic to export market volumes.

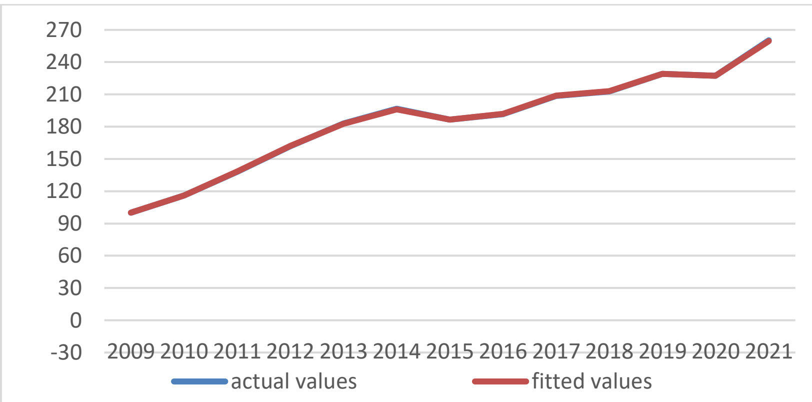

Figure 5. Actual and fitted values of total output in the non-oil sector

In the study, the CET function was also estimated for the service sector, and the results were as follows:

$$Y_j = 0.96 \cdot (0.29 \cdot E_j^{-0.29} + 0.71 \cdot D_j^{-0.29})^{-0.71}$$

R2=0.99, DW=1.99

The actual and fitted values of total output in the service sector, along with the dynamics of the difference between them, are presented in Graph 6.

Based on the estimated model parameters, the elasticity between service sector exports and domestically produced services sold in the domestic market is calculated as follows:

$$\sigma_{ser} = \frac{1}{1 + 0.29} = 0.41$$

The elasticity of substitution for the allocation of service sector production between domestic and foreign markets is 0.41, indicating that a 1% increase in the price of service sector products in the foreign market leads to a 0.41% increase in the ratio of domestic to export market volumes.

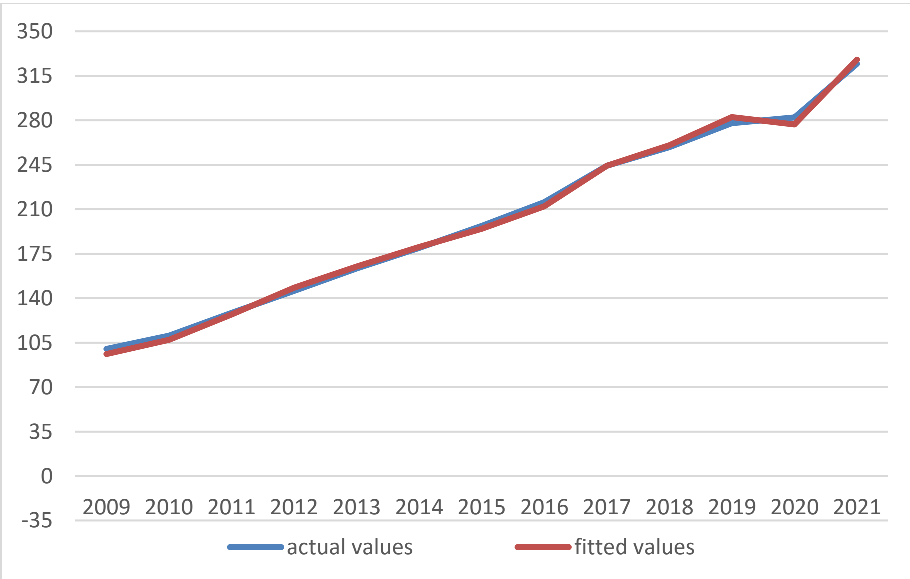

Figure 6. Actual and fitted values of total output in the service sector

DISCUSSION

Table 3 summarizes the estimated elasticities of the CET and Armington functions for all three sectors.

Table 3. Estimated foreign trade elasticities for the oil, non-oil, and service sectors

Sectors	Armington elasticities	CET elasticities
Oil sector	0.99	1.17
Non-oil sector	3.3	1.67
Services sector	0.99	0.41

Source: Authors' calculations

Table 3 presents the estimated Armington and CET elasticities for the oil, non-oil, and service sectors. For the oil and service sectors, the Armington elasticity is approximately 1, indicating unit substitution elasticity between local and imported goods. This means that during the considered period, changes in the relative prices of imported and domestic products were accompanied by proportionate changes in the ratio of their consumption. An elasticity not less than 1 also implies that these sectors face no significant barriers to imports.

In contrast, the non-oil sector shows a much higher Armington elasticity, equal to 3.3. This suggests a strong responsiveness of the import–domestic production ratio to price changes. For instance, when the global market price of a commodity declines, a larger share of domestic demand for that commodity is met through imports; conversely, when import prices rise, domestic production is able to substitute effectively for imports in meeting demand.

Regarding the CET elasticities in Table 3, the service sector's elasticity is below unity, reflecting the largely non-tradable nature of its products. For the oil sector, the CET elasticity is slightly above 1, indicating that export volumes increase at a marginally higher rate than price changes in foreign markets. The non-oil sector's CET elasticity, at 1.67, points to a substantial responsiveness of exports to relative prices. This may be partly attributed to policy measures aimed at enhancing the country's non-oil export potential.

Building on these sectoral results, it is noteworthy that aggregate estimates for the Azerbaijani economy reported by Devarajan, S.; Go, D.; Robinson, S. (2023) indicate Armington and CET elasticities of 0.5 and 0.36, respectively, which differ from our sectoral findings. This discrepancy may stem from the study period (1992–2018), especially the early years of independence, when producers' and consumers' responses to price changes were likely weaker due to limited foreign trade relations. Furthermore, as highlighted by Ahmad, S.; Montgomery, C.; Schreiber, S. (2021) and other studies, elasticities generally increase with higher sectoral disaggregation, which aligns with the higher values observed in our analysis. Overall, the estimated elasticities reveal notable differences across the oil, non-oil, and service sectors, providing valuable insights into the responsiveness of producers and consumers to price changes and trade opportunities. These findings can serve as an empirical basis for policymakers to design trade and production policies tailored to sector-specific characteristics.

CONCLUSIONS

In this study, Armington and CET elasticities for the oil, non-oil, and service sectors of the Azerbaijani economy were estimated using the Marquardt method in the Mathcad software package. This represents the first sectoral-level assessment for Azerbaijan and the first application of this method in the literature. The results provide important information on the foreign trade potential of the Azerbaijani economy and can be utilized in future trade and general equilibrium modeling. The methodology employed also allows estimation in cases where price statistics are unavailable, potentially facilitating similar assessments for other countries.

Conducting evaluations at a more disaggregated level and for countries with comparable economies, such as other oil-rich nations, and investigating the sources of observed differences, constitute promising directions for future research.

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Conceptualization: Yadulla Hasanli; Formal analysis: Yadulla Hasanli, Gunay Rahimli, Mattia Ferrari; Methodology: Yadulla Hasanli; Software: Yadulla Hasanli, Gunay Rahimli, Fuad Quliyev; Writing – original draft: Gunay Rahimli; Writing – review & editing: Fuad Quliyev, Mattia Ferrari

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